Fuzzy Lattice-Theoretic Operations over Data and Knowledge Structures

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Abstract

This explores the lattice-theoretic properties of the fuzzy processing of data and knowledge structures, such as First-Order Terms ($\mathcal{FOT}$s) and Order-Sorted Feature ($\mathcal{OSF}$) graphs, when ordered with endomorphic structure subsumption. Understanding the formal operational aspects of (fuzzy) structure unification and its dual (fuzzy) structure generalization are invaluable pursuits. Sets of declarative rules and axioms characterizing these operations as constraint normalization also provide efficient operational interpretations. For example, fuzzy $\mathcal{FOT}$ unification is used in Fuzzy Logic Programming ($\mathcal{FLP}$), thus adding fuzzy flexibility and expressivity to operations on $\mathcal{FOT}$s. This is true also for attributed instance and concept structures such as $\mathcal{OSF}$ data and knowledge representation and processing. We overview the extant state of the art in fuzzy unification of $\mathcal{FOT}$s (there is no known work on fuzzy generalization), stressing formal algebraic and operational specificities of other approaches. Our method so-doing is to investigate how to explain, and complete the expressivity of, existing approaches by explicating their data-structure approximation algebras (structure-preserving subsumption, unification, and generalization). Our objective is to extend to fuzzy operations (to "*fuzzify*") both lattice operations on $\mathcal{FOT}$s and $\mathcal{OSF}$ graphs. Calibrating sets of equations with fuzzy truth levels as approximation structures then exploits a fuller set of lattice-theoretic operations (fuzzy unification, but also fuzzy generalization). Our pragmatic motivation is that such fuzzy lattice operations on sets of $\mathcal{FOT}$ and $\mathcal{OSF}$ equations are very convenient in structured data and knowledge representation and processing, such as Fuzzy Information Retrieval. Why Lattice Theory? Because it is the mathematics of consistent approximation: fuzzy unification of $\mathcal{FOT}$ terms or $\mathcal{OSF}$ graphs being the Greatest Lower Bound (**GLB**) operation, and fuzzy generalization of $\mathcal{FOT}$ terms or $\mathcal{OSF}$ graphs being the Least Upper Bound (**LUB**) operation. This provides a fuzzy **GLB** operation over $\mathcal{FOT}$s and $\mathcal{OSF}$ object structures to act as, *e.g.*, a kind of "*fuzzy relational join*" to specify approximate retrieval patterns over a relational database. Dually, the **LUB** operation is the computation of the most specific $\mathcal{FOT}$ or $\mathcal{OSF}$ graph up to a fuzzy threshold that is their most specific approximate generalization for that truth threshold. Such could be be used, *e.g.*, for fuzzy schema inference or Machine Learning by fuzzy inductive reasoning.

Keywords: Approximate Information Retrieval; Lattices; First-Order Terms; Fuzzy Unification; Fuzzy Generalization; Fuzzy Feature Structures; Structure-Based Induction; Pattern-Directed Reasoning; Fuzzy Pattern Abduction; Fuzzy Pattern Induction; Fuzzy Machine Learning.

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